Aircraft and Rotorcraft Pilot Couplings – Tools and Techniques for Alleviation and Detection

ACPO-GA-2010-266073

Deliverable No. D2.4
Report Detailing Novel Pilot Models Developed for RPC Prediction

Contractual delivery date:
[March 2012]

Actual delivery date:
[May 2012]

Partner responsible for the Deliverable: UoL

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Dissemination level

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Executive Summary

Work Package 2 (WP2) deals with pilot and rigid-body vehicle modelling, and their application to the prediction of Rotorcraft Pilot Couplings (RPC). This deliverable first reports upon the latest progress made at the University of Liverpool and ONERA in a proposed set of optical-tau based criteria for the prediction of so-called Boundary Avoidance Tracking (BAT) events and Pilot-Induced Oscillation (PIO). The criteria has been developed during an initial investigation using in-flight and simulator test data for the roll-step boundary-tracking manoeuvre. The new work extends the research by applying the developed criteria by using data from a rotary wing aircraft simulated flight BAT pitch axis experiment. The investigation describes the experiment and shows that the proposed criteria are largely appropriate for the intended purpose. In particular, the criterion to predict a BAT PIO is shown to be valid. The work presented also contributes by showing why the variables involved in the criteria are important and relevant. Both the first and second derivatives of optical tau are shown to be surrogates for the pilot’s control activity in the axis considered. In addition, the investigation of the application of bifurcation theory on the joint pilot-rotorcraft simulation to expose the BAT phenomena is also covered in this deliverable.
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1. Introduction

One of the objectives of Work Package (WP) 2 is to develop new methodologies for pilot modelling to assist with the prediction and prevention of adverse Rotorcraft Pilot Couplings (RPCs). These will pave the way for the next step of the research which is to model the joint pilot-vehicle system for rigid-body RPC purposes. The key results of the pilot modelling exercise from the WP2 partners thus far are summarised in this Deliverable. It covers the development of the Boundary Avoidance Tracking (BAT) criteria based on optical tau information from the University of Liverpool (UoL) and the state-space form modelling of the joint pilot-vehicle for BAT RPC analysis by ONERA. The report expands the work of D2.3 which consisted of a review of current state-of-the-art pilot modelling methods.

Human pilot behaviour and rotorcraft/flight control system (FCS) dynamics are highly complex nonlinear systems that can ordinarily be treated using a three-pronged approach - analysis, computer simulation and experiment. Research conducted at UoL continues the investigation into BAT pilot-induced oscillations (PIOs). The initial study, which will be referred to in the remainder of the report as the ‘first phase’, was based upon data obtained from a simulated ‘Roll-step’ manoeuvre [1;2]. This manoeuvre was developed to evaluate lateral-directional handing qualities of rotorcraft, as an extension to the ADS-33 mission task element family [3]. As part of this research effort, a number of criteria that characterise both the pilot’s tracking performance and pilot inceptor workload have been developed to try to better understand the BAT phenomenon [1;4-6]. The next tranche of research (now termed the ‘second phase’), reported herein set out to test the predictive powers of these criteria using available data for a pitch-axis rotary wing task.

At the same time, ONERA have conducted some analytical investigations in the area of nonlinear RPC. The research is oriented towards developing analysis methods which prove that a given rotorcraft is sufficiently free from RPC proneness. The emphasis in this work is also on BAT RPC where the behaviour of a pilot in proximity to a boundary is the predominant factor [4]. The approach used is based on bifurcation theory and is founded on the analysis of the asymptotic behaviour of the closed-loop system that is made up of the pilot, the complete rotorcraft and its associated flight control system, when the parameters of the system are varying quasi-steadily. The system is augmented with a piloting task which provides the input signal to the system.

This Deliverable will report upon the results from these latest investigations.

2. Summary of Work Performed

As part of WP2, the UoL is required to develop new pilot model using pilot perception theories. Research conducted at UoL extended the application of optical tau into BAT events and PIOs [1;2]. The second phase, presented in this report, extends the work in the following key areas:

- The data within the first phase of the study in the roll axis did not contain any BAT PIOs, only BAT events. The study reported in this report does contain one such phenomena and so the criteria that could only be proposed in Ref. [2] is tested herein;
- The tau criteria to describe and predict BAT events and BAT PIOs developed for the roll axis of the aircraft are extended here into the aircraft pitch axis;
This report compares the timing parameters used to configure the Gray-BAT model from optical tau criteria with those obtained by other proposed approaches such as an optimisation method;

It is hypothesized in the field of visual perception that, in terms of a visual guidance strategy, the overall pilot’s goal is to overlay or close the gap between the perceived optical flow field over the required flight trajectory [7-9]. This report investigates the question arising as to how visually perceived information might influence the pilot’s control activity to allow him/her to guide the vehicle’s motion.

In WP2, ONERA also has conducted the investigation of application of the bifurcation method to the BAT RPC analysis problem.

2.1 Progress on Optical Tau in Relation to BAT Phenomena

The first-phase of the UoL BAT PIO research found that BAT-model parameters could be established using $\tau$ and $\dot{\tau}$ (see Appendix A). A strong correlation was shown to exist between the lateral control activity and the second-order derivative of tau ($\ddot{\tau}$). The deviations from the assumed $\dot{\tau}$ constant strategy are manifest in variations in $\ddot{\tau}$ for determining the BAT timing parameters, in contrast to the control acceleration variations proposed by others. The values of $\tau_b$ and $\dot{\tau}_b$ (here the subscript $b$ means the boundary) at some user-selected reference line, defined as the position where the pilot initiates the deceleration phase (e.g. the runway edge in Refs [1;2]), can be used to establish the potential for a BAT event/PIO.

The hypothesized conditions are summarized in Fig. 1. The reader is directed to Refs [1;2] for a more complete explanation of the derivation of these conditions.

The key result of the first phase of research was therefore to propose Fig. 1, although in a slightly different format, with slightly changed descriptors, as a means of either predicting the onset of or analysing, after the fact, a BAT situation.

![Fig. 1 $\dot{\tau}$ and $\ddot{\tau}$ conditions for BAT event and PIO prediction at the reference line [1;2]](image)

The procedure to apply the criteria shown in Fig. 1 as a post-processing tool is illustrated in Fig. 2.
Calculates the current $\dot{t}_b$ and $\dot{t}_s$ values of each point at the moment of crossing the reference line

Use $\dot{t}_u$ and $\dot{t}_d$ to find location in Fig. 3, and then to access likelihood of BAT event/PIO

$\dot{t}_b < 0.5$

Located in region 3?

No

Yes

$\dot{t}_b < 0$

No BAT Event Possible

No BAT Event/PIO

Located in region 5?

Yes

Calculate $\dot{t}_s$ values at $\dot{t}_b = 1$

$\dot{t}_s < 0$

Yes

BAT Event Definite

$\dot{t}_s < 0$

No

No

$\dot{t}_u > 0.5$ and $\dot{t}_u > 0$

($i$, $i+1$, $i+2$)

Yes

BAT PIO Highly Likely

$\dot{t}_u > 0.5$

No

BAT Event

Definite

End

Fig. 2 Flowchart for the proposed BAT event and PIO prediction approach

The application of the procedure in Fig. 2 is straightforward. A reference line is selected. This might, for example, be the object that the pilot initiates the deceleration phase to avoid (an impending boundary) or the object/parameter that the pilot is following using a point tracking strategy. For example, a runway edge was selected for the roll-step manoeuvre of the previous research activity. Once the parameter being tracked deviates from the reference, the next step is to calculate the instantaneous values of $\dot{t}_b$ and $\dot{t}_s$ from the current parameter value to the defined reference. The likelihood of a BAT event or BAT PIO occurring can be predicted based on the location of the result in Fig. 1. Of particular interest, however, is if the result of the analysis is contained within region 5 of Fig. 1. Because such results suggest that the aircraft is accelerating toward the boundary ($\dot{t}_b > 1.0$, though their $\dot{t}_s$ values are reducing ($\dot{t}_s < 0$)). Further information is required to know how rapidly $\dot{t}_b$ will decrease as the boundary is approached. The moment that is critical when describing the change between acceleration and deceleration towards a boundary ($\dot{t}_b = 1$) is therefore selected and analysed further. To avoid the method generating a large number of false positive BAT PIO predictions, the suggested convention to be adopted is to only flag a BAT PIO as being in progress when 3 consecutive data points are located in regions (4) and (6). This may be conservative but the user is free to modify this convention depending upon the specific configuration of an experiment.

2.1.1 Experimental Set-Up

The test data was obtained using UoL’s HELIFLIGHT flight simulator with a six degree-of-freedom (DOF) motion platform [10], shown in Fig. 3.
The flight dynamics simulation model used to generate the flight responses was a 6 degree of freedom nonlinear state-space Bo105 described in Ref. [6]. Its formulation is as shown in Eq. (3).

\[
\dot{x} = Ax + Bu + f \\
\dot{\phi} = p + (q \sin \phi + r \cos \phi) \tan \theta \\
\dot{\theta} = q \cos \phi - r \sin \phi \\
\dot{\psi} = (q \sin \phi + r \cos \phi) / \cos \theta
\]  

(1)

in which \( x = (u \ v \ w \ p \ q \ r \ \dot{p} \ \dot{q})^T \) and \( u = (x_u \ x_b \ x_c \ x_p)^T \). The state vector \( x \) contains the conventionally defined state variables for helicopter translational and rotational velocities in the \( x, y \) and \( z \) axis. The variables \( \phi, \theta, \) and \( \psi \) are the Euler angles. The control vector corresponds to longitudinal, lateral, collective, and pedal pilot controls. Finally, \( A \) and \( B \) contain the state and control derivatives and the term \( f \) adds the nonlinear contributions due to gravity, inertial couplings and flight dynamics couplings. The basic form of this model was considered as the ‘Baseline’ (Bas) model. In addition, the nonlinear state space model of Eq. (1) was augmented with a full-authority flight control system (FCS) to enhance its bandwidth for RPC prediction. This augmented model (Aug) was effectively a helicopter flight dynamics model with the effects of cross-coupling removed.

Fig. 4 illustrates the experimental configuration of the Head-Up-Display (HUD) generated using VAPS® with additional information of airspeed, pitch attitude, and altitude. The pilot was required to track the ‘oscillation director’ with the ‘aircraft bore-sight’ symbol. This is effectively a PT task. Two variations of this experiment were conducted: first (Type 1), the director symbol was driven using a sinusoidal oscillation between the two boundary lines;
second (Type 2), to try to add an element of uncertainty into the director symbol’s track, a ‘sum-of-sines’ trajectory was used to drive it. During the course of a given test point, the inter-boundary amplitude was reduced (in the sequence 14, 8, 5, and 3° visual angle). Furthermore, the frequency of oscillation (or base frequency in the sum-of-sines case) could be varied between test points. The following frequency values were used: 0.25, 0.375 and 0.5Hz.

Both the baseline and augmented models were utilized for the test campaigns analysed in this report. Both vehicle models were capable of having a time delay introduced into each control path channel. Time delay settings ($\tau_d$) of 0 ms, 100 ms, and 200 ms were tested. To add an element of boundary avoidance to the task, the pilot was instructed to keep the oscillation direction within the upper and lower oscillation boundaries. To try to ensure that this task was adhered to as strictly as possible, a ‘nuisance’ stall warning was played to the pilot after 5 boundary exceedances. As such, during each flight, the intention was for the pilot to track the director as aggressively as possible. After each test run, a PIO rating (PIOR) was awarded according to the PIO rating scale [11]. Pitch attitude results from two typical cases for each of these experiment types, along with the relative location of the boundary markers are shown in Fig. 5.

![Fig. 5 Illustration of the two types of pitch tracking cases tested](image)

The distance between the bore-sight marker and the approaching boundary defines the motion gap ($y$, negative) for the subsequent analysis. The motion closure velocity rate ($\dot{y}$) and acceleration rate ($\ddot{y}$) are defined to be positive when the boundary is being approached. Prior to reaching the boundary, the pilot must initiate a decelerative pitch control input and the focus of attention may change from the director to the boundary. Intuitively, the smaller the inter-boundary gap, the greater the likelihood that the pilot will have insufficient time to successfully manoeuvre within the boundaries and hence the lower would be the success of the task performance. If the pilot responds too late or reacts too aggressively, continuing attempts to fly the task may result in a BAT PIO. A study was initiated to assess the effect of boundary size on task performance. A measure of pilot performance was defined as the normalised root-mean-square error (NRMSE) between the director and bore-sight symbols shown in Eq. (2).

$$\text{NRMSE} = \frac{1}{A_b} \sum_{i=1}^{N} \sqrt{\frac{(y_d - y_b)^2}{N}}$$

(2)
where $A_b$ is the boundary pitch angle limit, $N$ is the number of data points sampled, $y_d$ is the pixel position of the director symbol, and $y_b$ is the pixel position of the bore-sight symbol. The results from the application of Eq. (2) to the datasets obtained are shown in Fig. 6.

![Figure 6](image)

**Fig. 6** Illustration of the variation in point-tracking performance as the boundary size is reduced

The results of Fig. 6 show the influence of the boundary size on the point tracking performance achieved. The generally larger NRMSE values from the Type 2 experimental results are to be expected due to the director trajectory being less predictable compared with the Type 1. For both types of experiments, the NRMSE values show a small decrease (except the Type 2 case with $\tau_d = 200$ ms) as boundary size decreases until reaching the boundary size = 5°. The tracking performance then degrades for the 3° boundary size. The same phenomenon also has been found in Refs. [12-14] for fixed-wing aircraft. This critical size (5°) represents the best achievable performance for the pilot in this experiment. For smaller boundary sizes, the pilot cannot improve tracking performance further and is unable to perform sufficiently rigorous tracking with the narrower boundary size. For an investigation into BAT Event and PIO phenomena, it is desirable that the pilot be able to perform the assigned PT task as successfully as possible. In this way, any switch between PT and BAT strategies is not corrupted by an inability to perform the PT task in the first place [14]. Therefore, the 5° boundary size task was considered the ideal candidate to expose the BAT phenomenon. As a consequence, only this task is considered for the remainder of the investigation in this report.

As an interesting aside, Fig. 6 shows that the control channel time delay ($\tau_d$) has significant influence on the NRMSE values. For example, the NRMSE values of the Type 1 cases are gradually increased by a factor of 10% when $\tau_d$ increases incrementally by 100 ms. Moreover, for Type 2, the values with $\tau_d = 200$ ms are 50% larger than the ones without a time delay. It is also clear, as would be expected, that larger control system delays result in higher PIOs. As a result, among the cases being investigated, the ones without time delay achieved the best performance. Moreover, a large delay can increase the BAT-PIO propensity, as reflected in Gray’s tracking tasks [4]. The results presented here are consistent with Gray’s findings in Ref. [15] and hence provide confidence in the data used in this report.

2.1.2 Assessment of the BAT event prediction criteria

Within the structure of the BAT model, Gray [12;13] and Warren [4] used different approaches to predict the characteristic timing points and propensity to BAT-type PIOs. An...
alternative approach shown in Fig. 1 and Fig. 2 in the first phase of the research has been developed to predict whether or not a BAT event or a BAT PIO exists [1;2]. A comparison between these two methods has been partially demonstrated using the Roll-step manoeuvre and has shown good consistency between them. This Section focuses on testing the criteria of Fig. 1 using the pitch tracking test manoeuvre introduced above to identify BAT events.

Before applying the criteria in Fig. 1 have been applied, three hypothesized possible scenarios that could develop during the two different types of pitch tracking task used are shown in Fig. 7.

For Type 1 cases, a successful tracking is shown as Scenario 2, with the trajectory of the bore-sight marker aligning smoothly with the director marker (the short dashed line in Fig. 7a), following a slight overshoot at certain periods. Moreover, the maximum amplitude of the director symbol is the same as the boundary. Therefore, when reaching the peak amplitude points of the lead signal, the pilot should always be aware of the existence of the boundary. This means that the Type 1 experiment could be generally expected to generate BAT events.

Scenario 2 is a PT with partial failure, where the pilot is unable to track the director symbol for short periods but does not exceed the boundaries. For Scenario 3, the pilot progressively loses PT performance and the manoeuvre diverges beyond the assigned boundaries. Finally, a BAT PIO develops. In the results from the trial series drawn on for this report, examples of the three scenarios described above have been shown.

For Type 2 cases shown Fig. 7b, Scenario 1 represents a successful tracking, with the trajectory of the bore-sight marker aligning smoothly with the centreline (the director marker), also following a slight overshoot. The pilot’s focus of attention remains the centreline tracking throughout the manoeuvre and there is no disruption from the proximity of the outer boundaries. Scenario 2 is a BAT event, where the pilot significantly overshoots the centreline and switches attention to the outer boundary. The pilot successfully brings the trajectory back within the boundary and switches attention to successfully tracking the centreline; this situation is described as a BAT event since the pilot switched from point to BAT but successfully re-engaged with the PT task. For Scenario 3, the pilot fails to re-establish PT and, instead, the manoeuvre diverges and develops into a BAT PIO between the adequate-performance boundaries.

In the results from the trial series drawn on for this report, examples of the three scenarios described above have been shown. In this Section, six cases of the first two scenarios, three from each type of tracking test, were selected to test the BAT event prediction criteria.

The criteria in Fig. 1 have been applied, using the process shown in Fig. 2, to each of the sets of experimental data. Three Type 1 cases (all with a 5° boundary) are shown in Fig. 8 to illustrate the analysis process.
It can be seen by inspection in Fig. 8 that point tracking performance degrades as the time delay is increased, the number of boundary exceedances increasing with increased time delay. Moreover, two points need to be raised here. First, for the Type 1 cases, as shown in Fig. 5a (or Fig. 8) and mentioned above (scenarios in Fig. 7a), the Type 1 experiment could be generally expected to have BAT events. Second, there is no BAT PIO shown in the example cases of Fig. 8 (SCENARIO 3, Fig. 7a). Therefore, the results from testing the criteria in Fig. 1 are also expected to show these two points. In addition, to follow the suggested procedure in Fig. 2, it is first necessary to determine a reference line that it is hypothesized that the pilot uses to initiate the deceleration phase of the tracking manoeuvre. This crossing of this reference triggers the calculation of $\tau$ and $\tilde{\tau}$. Because of the symmetry of the manoeuvre from one boundary to the opposite, the centreline would be a natural selection by the pilot to start the deceleration. In addition, this selection is consistent with the results from the roll-step manoeuvre [1;2] in which the pilot was found to sub-consciously use the runway centreline to initiate the deceleration. If each of the cases of Fig. 8 are taken in turn, the $\tau$ and $\tilde{\tau}$ values at the centreline reference for each of these Type 1 cases are plotted in Fig. 9.

Fig. 8 Illustration of three Type 1 test cases to test the BAT event prediction criteria
Fig. 9 (left): $\ddot{\tau}$ and $\dddot{\tau}$ values plotted against the developed criteria for Type 1 cases; (right): for those points with $\ddot{\tau} > 1.0$ in (a, c, and e)

The $\ddot{\tau}$ and $\dddot{\tau}$ values in Fig. 9 are mainly distributed in regions 3 and 5. With regard to the criteria in Fig. 1, these points indicate definite BAT events, which is consistent with the first point raised above that the experiment configuration will generally show these kinds of phenomena when tracking up to the boundary itself. Moreover, for those points with $\ddot{\tau} > 1.0$ in region 5, they have negative $\ddot{\tau}$ values except for circle symbol (9) when $\ddot{\tau} = 1$ shown in Fig. 9f. This implies that the motion is decelerating even after $\ddot{\tau} < 1$. For example, although undergoing two boundary exceedances between squares (9) and (11) in Fig. 8b, the trajectory for Case 2 finally recovers the tracking task after square (11). This can be explained by the $\ddot{\tau}$ and $\dddot{\tau}$ values for the squares (9), (10), and (11) in Fig. 9c. The square (9) has $\ddot{\tau} = 1.1$ and $\dddot{\tau} = -1.5$, or simply $(1.1, -1.5)$. These values indicate the motion is accelerating toward the boundary with a decreasing acceleration rate. Therefore, the motion at this moment is predicted to be a highly probable boundary exceedance and a possible BAT-PIO. Afterwards, square (10) with $(0.98, -0.89)$ also theoretically suggests that the motion would impact the boundary with a residual velocity but that a BAT-PIO becomes less possible. Finally, the square (11), with $(0.26, -0.17)$, means the boundary exceedance problem has been overcome and the BAT PIO is now impossible at this crossing moment. The analysed results are consistent with the actual situation. The same explanation can be given for the more severe Case 3 (e.g., the circles 8, 9, and 10) why there is no BAT-PIO shown. These results have reached same agreement with the second point raised above that there is no BAT PIO shown in these three cases. Therefore, the criteria in Fig. 1 have been successfully tested.

Additionally, there are nine data points located in regions 1 and 2. The eight points except the square (5) (located in region 2) have large negative $\ddot{\tau}$ values that theoretically do not lead to BAT events. This theoretical point is reflected by their related consequent movements.
stopping before the boundaries in Fig. 8. Moreover, its movement finally overshoots the boundary as shown in Fig. 8b, as indicated theoretically by the positive $\dot{\tau}$ value ($\dot{\tau}$ is increasing when approaching the boundary). All these results presented show the validation of these criteria in Fig. 1.

For the Type 2 experimental data, the director signal forms the reference for the analysis. The pilot’s primary task is to track the director’s trajectory and to avoid pitch excursions towards either (upper or lower) boundary. The points at which the reference line is crossed for three Type 2 test points cases (each with a 5° boundary) are illustrated in Fig. 10.

Fig. 10 Illustration of three Type 2 test cases to test the BAT event prediction criteria

After checking the trajectory shown in Fig. 10 and then comparing these with the scenarios in Fig. 7b, similarly to the three Type 1 cases, two points are also raised here. First, the tracking performance of these three cases is generally good. A few parts of the trajectories resemble PT, the first scenario in Fig. 7b. Second, it is clear that, whilst tracking performance is not perfect, no boundary exceedances occur. Hence, three cases in Fig. 10 contain no BAT PIOs. Therefore, for the validity of Fig. 1, the final testing results need to reflect these two points.

The associated $\dot{\tau}$ and $\ddot{\tau}$ values are plotted in Fig. 11, within the regions defined as per Fig. 1 in order to assess the efficacy of the BAT event prediction criteria.
Now the criteria in Fig. 1 are tested here. First, as shown in Fig. 11, all the crossing points are located within regions 1, 3, and 5 and there are no points located within regions 4 and 6. According to Fig. 1, this kind of distribution indicates a lower probability for a BAT PIO. Although some points lie in region 5 ($\tau > 1$) that suggest that a BAT PIO will be more likely, there are free of BAT PIO. There are two main reasons. First, inspection of their related $\ddot{\tau}$ values shows that they are large and negative. This indicates that $\ddot{\tau}$ will quickly decrease. Second, the $\ddot{\tau}$ values at $\ddot{\tau} = 1$ in Fig. 11(right) for those with $\tau > 1.0$ in Fig. 11(left) are also all large negative. For example, the circle (5) in Fig. 11e with (2.1, –19.1) indicates accelerating toward the lower boundary with a significant $\ddot{\tau}$ decreasing rate. Further checking shows that $\ddot{\tau} = –12.6$ when $\ddot{\tau}$ reduces to be one in Fig. 11f. This implies that the motion is still undergoing quick deceleration even after $\ddot{\tau} < 1$. Theoretically speaking, this indicates that the pilot can highly probably stop before the boundary. This analysis is consistent with the practice situation in Fig. 10 that the bore sight initially deviates from the centreline and later it is brought back tracking again before reaching the lower boundary. This represents a BAT event. Therefore, the second point raised above that no BAT PIO is contained in these three cases is successfully tested by the criteria in Fig. 1.

Second, the points in region 3 ($0.5 < \ddot{\tau} < 1$) also represent BAT events, by following the similar reasons given above. Furthermore, the points in region 1 stand for no BAT events. For instance, the diamond case (1) with (0.05, –3.8) in Fig. 11a shows the motion can stop before the boundary according to Table A1. Therefore, this part of the trajectory between points 1 and 2 in Fig. 10a is considered as to be a PT process, following along the centreline. The similar explanation can be applied on other points in region 1. These findings again confirm the first point given above. As a consequence, the criteria in Fig. 1 are thus successfully tested on the selected three cases.
2.1.3 Application of Optical Tau to Predict BAT PIOs

In the first phase of this research, no BAT PIO events were observed in the roll-step data. However, one case of a suspected BAT PIO was recorded for the pitch tracking task. In this Section, the criteria of Fig. 1 for a BAT PIO are tested against these data. The bore-sight symbol and the longitudinal inceptor control positions for this test point are shown in Fig. 12.

The time history is divided into five periods for convenient analysis based on the peak points of the bore-sight position on the HUD. As shown in Fig. 12a, the boundary is successfully avoided though the director symbol is not well tracked during and at the end of period A. Moreover, the pilot introduces a large input just before reaching the boundary (around 44.5 s) in order to track the director. The director trajectory is crossed at 45 s in period B but the applied control is so large that the bore-sight marker is driven beyond the opposite boundary. After this time, it is presumably the pilot’s intention to bring the bore-sight symbol back within the defined boundaries. Instead, a sustained oscillation that can be characterised as a BAT PIO ensues, as shown in periods B to E. Now applying the optical tau approach of Fig. 1 and Fig. 2 provides results for this case shown in Fig. 13.

Fig. 12 Illustration of a potential BAT PIO case in the pitch axis

Fig. 13 (a): $\tau$ and $\dot{\tau}$ values plotted against the developed criteria for BAT PIO case; (b): for those points with $\dot{\tau} > 1.0$ in (a)
The $\dot{\tau}$ and $\ddot{\tau}$ values (diamond symbols) calculated at the centreline are all located in region E in Fig. 13a. Their corresponding $\dot{\tau}$ values (circular symbols) at $\tau = 1$ are also shown in Fig. 13b. Only the first reference crossing point in period A has a negative $\dot{\tau}$ and the other four cases are positive. By following the same analysis as in the previous Sections, this indicates that a BAT event is present after the first crossing moment, and that the subsequent motions have a high probability of developing into a BAT-PIO situation. The theoretical results are consistent with the typical response presented in Fig. 12. As such, the criteria for region (5) with respect to a BAT PIO appear reasonable.

In addition, to better understand and illustrate the development of the BAT-PIO phenomenon, the analysis process is broken down in Fig. 14. The results from periods D and E replicate period C and are consequently ignored here.

**Fig. 14 Analysis of BAT PIO with optical tau information**

As shown in Fig. 12b and Fig. 14d, the pilot abruptly increases the control effort at $t = 44.5$ s, presumably to try to capture the director marker and to avoid the impending boundary ($\theta = 5^\circ$). As a consequence, the $\dot{\tau}$ value in Fig. 14d turns to become negative again at the later part of period A and $\dot{\tau}$ in Fig. 14g continues decreasing at a larger rate that finally reduces to be less than 0.5. This also is reflected with the non-zero value of the $\tau$ curve in Fig. 14g. As a result, the motion ends without overshooting the boundary. Therefore, period A represents the occurrence of a BAT event and this is consistent with the prediction from Fig. 13.

However, because of aggressive control of the cyclic stick at the end of period A in Fig. 14d, the pilot has to immediately push the stick forward at the beginning of period B shown in Fig. 14e, in order to bring down the pitch attitude to capture the director symbol. Hence, the control is so aggressive that at the crossing moment within period B in Fig. 14b, $\dot{\tau} = 1.3$, which indicates an acceleration towards the lower boundary. Meanwhile, as shown in Fig. 14e, $\dot{\tau}$ is first negative and then quickly becomes positive and continues to increase monotonically. This results in that the $\dot{\tau}$ value in Fig. 14h increases significantly just after reaching the minimum value around one. This means that the motion is accelerating toward to the boundary ($\theta = -5^\circ$). As also reflected in Fig. 13b, the $\dot{\tau}$ value at $\dot{\tau} = 1$ (circle 2) is a large positive value. Therefore, the boundary is overshot and a further large opposite control input [266073_ARISTOTEL_D2.4_Report_Detailing_Novel_Pilot_Models_].
is required within period C to bring the aircraft trajectory back between the boundaries. With regard to the criteria listed in Fig. 1, these indicate that the occurrence of a BAT PIO is highly possible. Furthermore, the same analysis can be given to period C with (1, 1.94) in Fig. 14i, and Fig. 14f. This again implies the BAT PIO continues developing. All these results indicate that a severe BAT PIO is occurring and the BAT PIO criteria of Fig. 2 appear correct for this kind of event.

2.1.4 Characterisation of a BAT Event and a BAT PIO

The results presented in the previous Section show that the criteria developed can predict the occurrence of BAT events in the pitch axis. In this Section, these BAT events will be characterised quantitatively — their initiation \((\tau_{bs})\) and end \((\tau_{be})\). Moreover, the results obtained from both tau theory and Blake’s optimisation technique [12] for the Gray model are compared to show the effectiveness of the optical tau approach.

For optical tau, the variable \(\tau_{bs}\) is determined at \(\dot{\tau}_b = 1.0\), by following the approach developed in Refs. [1;2]. The \(\tau_{bs}\) variable was defined in the first phase of the research to be the same variable \(\tau_{max}\) in Gray’s model that characterises the moment when the pilot applies maximum control input. However, a BAT event may end before the maximum control input is reached or may continue after the maximum input is made. Therefore, the variable \(\tau_{be}\) is redefined in this report and is selected as the timing when \(\dot{\tau}_b < 0.5\). This is because, as shown in Table A1, the motion will stop before the boundary when \(\dot{\tau}_b < 0.5\) during the deceleration phase. As for Blake’s technique, \(\tau_{bs}\) and \(\tau_{be}\) variables are obtained through an optimisation method on a modified BAT pilot structure [12]. This approach is summarised as follows.

At first, a bias terms \((\Delta_{bias})\) that accounts for the process noise has been added into the formula of Gray’s model as shown in Eq. (5),

\[
K = \frac{\tau_{min} - \tau_{b}(1 - \tau_{bs})}{\tau_{min} - \tau_{max}} K_m + \Delta_{bias} \tag{3}
\]

Though the appearance of Eq. (3) is simple, its structure is highly nonlinear e.g. the nonlinear product term \(- \tau_{min} K_m\). Therefore, to simplify the identification procedure, the time delay \((\tau_{dp})\) is obtained by checking the time difference between the input \((\tau_b)\) and the output \((x_b)\). Second, the derivative-free Nelder-Mead search algorithm [16] is adopted in this report and the cost function is constructed as

\[
J = ||K - x_b|| \tag{4}
\]

The results of \(\tau_{bs}\) and \(\tau_{be}\) variables from both approaches applied to the three Type 1 cases are compared in Table 1.

<table>
<thead>
<tr>
<th>Case 1: 0.0</th>
<th>(\tau_{bs})</th>
<th>(\tau_{be})</th>
<th>(\tau_{min})</th>
<th>(\tau_{max})</th>
<th>(\tau_{dp})</th>
<th>(K_m), %</th>
</tr>
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<tbody>
<tr>
<td>0.082</td>
<td>0.021</td>
<td>0.11</td>
<td>0.09</td>
<td>9.0</td>
<td>3.3</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2: 100</th>
<th>(\tau_{bs})</th>
<th>(\tau_{be})</th>
<th>(\tau_{min})</th>
<th>(\tau_{max})</th>
<th>(\tau_{dp})</th>
<th>(K_m), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.064</td>
<td>0.021</td>
<td>0.11</td>
<td>0.15</td>
<td>7.1</td>
<td>2.9</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3: 200</th>
<th>(\tau_{bs})</th>
<th>(\tau_{be})</th>
<th>(\tau_{min})</th>
<th>(\tau_{max})</th>
<th>(\tau_{dp})</th>
<th>(K_m), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.031</td>
<td>0.041</td>
<td>0.065</td>
<td>8.8</td>
<td>4.5</td>
<td></td>
</tr>
</tbody>
</table>
The mean values of $\tau_{\text{min}}$ and $\tau_{\text{max}}$ from the Nelder-Mead algorithm shown in Table 1 for three cases are quite close. They are also approximately equal to the corresponding ones obtained from optical tau ($\tau_{\text{be}}$ vs $\tau_{\text{min}}$ and $\tau_{\text{be}}$ vs $\tau_{\text{max}}$). This reflects the possible utility of the optical tau approach. Furthermore, their standard deviations (STDs) are within 10% except $\tau_{\text{max}}$ at $\tau_d = 100$ ms and 200 ms. The mean $\tau_{\text{dp}}$ values are found to be close to their effective delays (introduced inceptor delay plus the 40 ms transmission delay in the simulator). The large STDs for the pilot maximum gain ($K_m$) are expected due to the different control efforts that are involved during different BAT events. Finally, three inputs, each from one case, constructed with parameter values in Table 1 have been plotted in Fig. 15.

![Fig. 15 Comparison of $x_b$ with Gray model output with identified values](image)

The cases illustrated in Fig. 15 show the constructed input from the Gray model with both the identified and optical tau values generally captures the trend of the recorded longitudinal cyclic input. This represents, to some extent, the fidelity of the identified values with the optimisation method used. Moreover, the good fit also shows the applicability of optical tau approach to determine the key BAT parameter values.

The same optimisation approach was also applied to the BAT-PIO case analysed above. Its identified model parameters are: $\tau_{\text{min}} = -0.53$ s, $\tau_{\text{max}} = -0.16$ s, $\tau_{\text{dp}} = 0.23$ s, and $K_m = 37$ %. Similarly, $\tau_{\text{bs}} = -0.55$ s, and $\tau_{\text{be}} = -0.13$ s, obtained from optical tau for period A. These values are quite close to those shown in Table 1. This may indicate these values reflect the pilot’s ability to respond to an impending boundary, as suggested by Gray [4;17].
constructed inputs with these optical tau values as well as the optical tau information have been presented in Fig. 16.

![Graph](image)

**Fig. 16 Optical tau and the constructed inputs during the BAT PIO**

Each spike in Fig. 16a corresponds to the individual period in Fig. 12a, respectively. The negative $\tau_b$ values in period A (below 44.7 s) reflect successful boundary avoidance and represent a BAT event, as discussed above. Moreover, the constructed input has a reasonably good agreement with the actual pilot inputs, shown in Fig. 16b. In addition, the typical square shape of the constructed inputs, again fits well the longitudinal cyclic input ($x_b$) from periods B to E, indicating the occurrence of the BAT PIO [4] triggered by the approach of the impending boundary during period B. Moreover, the $\tau_b$ values within each period (from B to E) increase sharply and finally reach the positive infinity as shown in Fig. 16a. This also suggests that the pilot operated beyond the defined boundary in each period, as reflected by the results presented in Fig. 12a. These results show the potential for the effective application of the Gray model for offline prediction of BAT events PIOs.

### 2.1.5 Relationship between Control Activity and Optical Tau

The available $\dot{z}$ information has been extensively exploited in Fig. 1 and Fig. 2 and referred to during the analysis in the previous Sections. The relationship between the pilot input and optical tau was the subject of a preliminary investigation in the first phase of the research for the Roll-step manoeuvre [1;2]. It was shown that the vehicle’s trajectory was well matched to a theoretical $\tau$-guided strategy. In addition, the deviations from a constant $\dot{z}$ guidance strategy were reflected in variations in $\dot{z}$, indicating a close correlation between control movements and $\dot{z}$. However, the question still remains as to how perceived information can be used to guide the motion and affect the control activity in this manner. This Section explores this relationship in more detail using the pitch attitude tracking task as a case study.

The use of $\dot{z}$ as a measure of deviation from the perfect tau guided motion can be explored through the expression defined as follows,
\[ \ddot{\tau} = -\frac{y^3 y + y y y y - 2 y y^2}{\dot{y}^3} \]  
Eq. (5) can be further expressed as follows,

\[ \ddot{\tau} = \frac{r_i}{r_j} \left( \frac{1}{r_i} - \frac{2}{r_j} + \frac{1}{r_j} \right) \]  

in which \( r_i = \frac{\dot{y}}{y} \). To ascertain the ‘correlation’ between control movement and \( \ddot{\tau} \) in the pitching tracking test, four examples have been selected from the data and are shown in Fig. 17.

![Fig. 17 Comparison of longitudinal cyclic control and \( \ddot{\tau} \) of the pitch tracking test](image)

The values in Fig. 17 have been normalised by their own absolute maximum amplitude in the selected period. Moreover, the additional 200 ms compensation has been added to the \( \ddot{\tau} \) values in (c, d) in order to make the comparison easier. It can be seen that the shapes of the calculated \( \ddot{\tau} \) line generally capture the variation of the longitudinal inceptor input. Moreover, the correlation is more evident at the later part of the plot that is the deceleration phase when approaching the boundary. In addition, the correlation coefficients of the shapes of most cases being investigated were larger than 0.8 (or < –0.8) which indicates the strong linear relationship of the directions between two variables [18]. Therefore, the close correlation between \( \ddot{\tau} \) and control activity presented in the roll-step manoeuvre [1;2] are also evident in the pitch tracking manoeuvre.

This correlation can be explained by the tau theory in that it hypothesizes that pilots respond to the consistent and continuous information available in the visual flow field to apply the proper control strategy to close any perceived motion gaps [19;20]. Optical tau suggests that BAT manoeuvres are flown primarily with reference to outside world visual cues, and the associated tau information [1;2;7]. As hypothesized, the main control strategy is to overlay the actual flight path with the desired flight path. If the pilot perceives a gap between these
two flight paths, further corrective controls will immediately follow. Meanwhile, the control activities have to fit both spatial and temporal constraints. Therefore, closing a motion gap with a constant \( \dot{\tau} \) strategy implies a smoothly varying control action that brings the aircraft to the desired state without over-control or over-shoot. However, in practice, the pilot never follows the guide perfectly and the smooth gap closure is frequently perturbed by high-frequency fluctuations and low-frequency drift. In turn, the pilot will apply corrective action and both state and control deviations will, in principle, be related to fluctuations in \( \dot{\tau} \), and will therefore be manifested by variations in \( \dot{\tau} \).

This correlation provides a possible way to explain the hypothesis of \( \tau \)-guidance control strategies [7;20]. In the previous investigations [19;20], it is hypothesized that \( \tau \) and \( \dot{\tau} \) are the variables that the animals and humans might use to control their motion and humans to control aircraft. For example, a helicopter pilot will stop before the obstacle when \( \dot{\tau} < 0.5 \) provided that a constant deceleration \( \tau \)-guide is followed [7]. It is also assumed that optical tau information is directly perceived to achieve prospective control of the motion, working analogously as a feedforward controller [21]. However, if these hypothesized points are to be investigated, the difficulty of directly relating the optical information (\( \tau \) and \( \dot{\tau} \)) to consequent control activity will immediately arise because of a lack of knowledge about the physical connection between them.

This report addresses this difficulty by working on the variable \( \ddot{\tau} \) instead of control activity because of their close correlation. Therefore, the problem reduces to an investigation as to whether or not variations in \( \tau \) and \( \dot{\tau} \) contribute significantly to variation in \( \ddot{\tau} \). The sensitivity analysis is carried out on Eq. (5) and Eq. (6) to explore the significance of each variable that contributes to the \( \ddot{\tau} \) variable and to facilitate understanding as to how the relationship between perceived information (\( \tau \), \( \dot{\tau} \), \( \ddot{\tau} \), and control activity can be bridged.

The suggested procedure has been applied on four examples in Fig. 17. The results from Fig. 17c are plotted in Fig. 18 for illustration. Similar results are also found for the other three examples and therefore are not included here for the sake of brevity.

![Sensitivity Values](image1)

**Fig. 18 Sensitivities of \( \ddot{\tau} \) with respect to related variables**

Fig. 18a shows the motion gap (\( y \)) and the gap closure rate (\( \dot{y} \)) have significant influence on the \( \ddot{\tau} \) variable as compared to the others (\( \ddot{y} \) and \( \dddot{y} \)) that have a reasonably small variation. Therefore, the shape of the \( \ddot{\tau} \) variable is primarily determined by the two variables (\( y \) and \( \dot{y} \))
that are used to define \( \tau \) in Eq. (A1). This indicates that tau information \((\tau)\) may play a vital role in shaping the pilot’s control inputs, taking into account the strong correlation between \( \dot{\tau} \) and control activity, shown above. This point can be further asserted by the high sensitivity of \( \tau_y \) and \( \tau_z \) on the \( \dot{\tau} \) variable shown in Fig. 18b. This is particularly true when the helicopter enters into the deceleration phase (after the normalised time point 0.45). The spike in Fig. 18b corresponds to the moment when \( \dot{\gamma} = 0 \). After this moment, the sensitivity of \( \tau_y \) is almost negligible. Furthermore, \( \dot{\tau} \) is defined by depending purely on \( \tau_y \) and \( \tau_z \) with regard to Eq. (A1). Hence, the high sensitivity of \( \tau_y \) and \( \tau_z \) on the \( \dot{\tau} \) variable (that has strong correlation with control activity) also indicates the possible strong connection between \( \dot{\tau} \) and control activity. Therefore, this is a good example of support for the hypothesis about the use of a \( \dot{\tau} \) control strategy in the deceleration stage of a motion [7]. The results presented here show the influence of optical tau information on the pilot’s control activity. These findings further support why tau information can be used to model pilot control strategies and hence predict BAT events and BAT PIOs.

2.2 State-Space Form Modelling of the Joint Pilot-Vehicle for BAT RPC Analysis

The bifurcation method requires the pilot-aircraft system and the piloting task to be formulated into a nonlinear dynamical model represented by ordinary nonlinear differential equations:

\[
\dot{X} = F(X, \lambda)
\]

where,

- \( X \) is the \( n \)-dimensional state vector
- \( \lambda \) is the \( m \)-dimensional parameter vector
- \( F \) is a vector of \( n \) nonlinear continuous and differentiable functions.

This state-space form modelling is presented hereafter through the formulation of the equations of the different subsystems: rotorcraft/FCS model, piloting task, point tracking (PT) control, boundary avoidance tracking (BAT) control, switching logic (see Fig. 19).
Fig. 19 Joint pilot- Helicopter/FCS model

The model of the helicopter and its FCS is defined by the following nonlinear system:

\[ \dot{x} = f(x,u) \]

where

- \( x \) is the state vector of the helicopter and FCS
- \( u \) is the control vector of the pilot inputs
- \( f \) is a vector of nonlinear functions.

The parameters of the model are the FCS time delay, saturation and rate limits.

Piloting task

The selected piloting task is a roll step manoeuvre followed by the stabilization of the ground track of the helicopter trajectory inside boundaries materialized by markers aligned on the ground. An illustration of the manoeuvre is shown in Fig. 20, where \( Y \) and \( X \) are the lateral position and longitudinal position of the helicopter, and \( \tau_e \) is the time to the boundaries. The adequate performance is represented by the red lines and the desired performance is represented by the blue lines.
Point tracking control

The PT roll step control system is formulated through its kinematics, outer-loop guidance law and inner-loop control law.

The kinematics are expressed as

\[ \dot{y} = g(x, y) \quad y \rightarrow y_d \]

where \( y \) is the helicopter lateral position and \( y_d \) is the desired lateral position.

The outer-loop PT guidance law is a required bank angle, function of \( x, y, \) and \( y_d \):

\[ \phi_{PT\_required} = -\tan^{-1}\left[ \frac{2\zeta\omega V \sin \chi + \omega^2 (y - y_d)}{g \cos \chi} \right] \]

where \( V \) and \( \chi \) are the helicopter velocity and heading angle, \( \omega \) and \( \zeta \) are the undamped natural frequency and damping ratio of the guidance law.

The inner-loop control law is a function of the helicopter state vector and the required bank angle: \( u(x, \phi_{PT\_required}) \).

The parameters of the PT model are the gain of the guidance law and the pilot time delay.

Boundary avoidance tracking control

In the same manner, the BAT roll step control system is formulated through its kinematics, outer-loop guidance law and inner-loop control law.

The kinematics are expressed as
\[ \dot{y} = g(x, y) \quad y_{\text{min}} < y < y_{\text{max}} \]

where \( y \) is the helicopter lateral position and \( y_{\text{min}} \) and \( y_{\text{max}} \) are the boundaries.

The outer-loop BAT guidance law is a required bank angle, function of the time to boundaries:

\[
\phi_{\text{BAT, required}} = -K \left( \frac{y - y_{\text{max}}}{g(x, y)} \frac{y - y_{\text{min}}}{g(x, y)} \right) \phi_{\text{max}} \text{sgn} (\chi)
\]

The gain of the guidance law is given by the following expressions [4]:

\[
K = 0 \quad t_{\text{min}} < \tau_e
\]

\[
K = \frac{t_e - t_{\text{min}}}{t_{\text{max}} - t_{\text{min}}} K_m \quad t_{\text{max}} \leq \tau_e \leq t_{\text{min}}
\]

\[
K = K_m \quad \tau_e < t_{\text{max}}
\]

where \( \tau_e \) is the time to the boundaries and \( t_{\text{min}}, t_{\text{max}} \) and \( K_m \) are the parameters of the BAT model [2].

The inner-loop control law is a function of the helicopter state vector and the required bank angle: \( u(x, \phi_{\text{BAT, required}}) \)

By combining respectively the states and the parameters of the above subsystems into an augmented state vector and an augmented parameter vector, the state-space form of the closed-loop pilot-vehicle system can be expressed as:

\[
\dot{X} = F(X, \lambda)
\]

The next step of the investigations will be the prediction of the stability of the closed-loop system using nonlinear analysis tools. It is expected that BAT RPC would be related to the existence of limit cycles.

Fig. 21 to Fig. 23 show the results of a simulation of the joint pilot-IAR330 Puma model in a BAT roll step manoeuvre for a given combination of the BAT parameters. The results display the ground track of the helicopter trajectory, the time to the boundaries, the helicopter state vector (body velocities, angular rates and attitude) and control vector (collective, longitudinal cyclic, lateral cyclic and tail collective).
Fig. 21 Simulation of the joint pilot-IAR330 Puma model in a BAT roll step manoeuvre

Fig. 22 Results from simulation of the joint pilot-IAR330 Puma model in a BAT roll step manoeuvre
Fig. 23 Results from simulation of the joint pilot-IAR330 Puma model in a BAT roll step manoeuvre

The simulation plots show a case of the closed-loop system going into a limit cycle, the combination of the system parameters being in this case:

PT guidance law:

$$\omega = 0.314 \text{ rad/sec}$$

$$\zeta = 0.75$$

BAT guidance law:

$$\phi_{\text{max}} = 45 \text{ deg}$$

$$K_m = 0.50$$

$$t_{\text{min}} = 1.5 \text{ sec}$$

$$t_{\text{max}} = 1.0 \text{ sec}$$

This behaviour will be predicted by bifurcation theory and verified by simulation in the framework of Task 2.4.
3. Results and Conclusions

This report presents the results of the progresses made in both UoL and ONERA. First, the main conclusions of the UoL’ study are listed as follows:

- In line with previously reported results, the tracking performance for a point tracking task being operated between a set of boundaries that can be obtained from a pilot can be improved as the boundary size is decreased up to a point. However, a limit will be reached whereby further decreases in boundary size will cause a deterioration of tracking performance. For this study, the optimum boundary size was found to be for the 5° test point.

- For cases of suspected BAT events, the data available primarily lay in region of the criteria where \( \tau < 0 \) and specifically in regions (1), (3) and (5). For each of these regions, the predictive capability of the criteria appears to be good and the descriptors appropriate. This is also true for the one data point where \( \tau > 0 \) but for such limited data, no firm conclusions can be drawn.

- For one case of a suspected BAT PIO, the proposed criteria also appear appropriate, although selecting the \( \dot{\tau} = 1.0 \) location may be a more effective strategy in this case.

The first phase of the research raised a number of issues that have been investigated in this report. First, a BAT event can be effectively characterised by optical tau in the roll-step manoeuvre. Further experiments are required to extend this investigation to other aircraft types and manoeuvres. Second, the criteria that have been developed and presented rely heavily on the first and second derivatives of the optical parameter tau. It is not immediately obvious why this optical parameter might be involved in the development of BAT events and PIOs. The main conclusions of the second set of investigations are summarised as follows.

- Optical tau information has been applied in a pitch tracking task to indicate the initiation and end of a BAT event. The BAT event initiation timing values, achieved from optical tau and Gray model are very close.

- This report shows that the pilots inceptor inputs are closely correlated with the \( \dot{\tau} \) during the manoeuvre and that therefore \( \dot{\tau} \) is a surrogate for the control strategy. It is further shown that the \( \ddot{\tau} \) information that it is posited that the pilot can perceive also strongly influences the subsequent control activity. As such, it should be of no surprise that these two parameters have an important role to play in predicting when that control activity will result in a BAT event of PIO.

The first and second phases of this investigation have been carried out on data not intended specifically for the purpose to which it was eventually put. As such, a further experiment has been planned to assess the criteria further and in particular those regions of the criteria that have not yet been fully explored by this report. In addition, the efficacy of potential near real-time early warning systems for BAT events and PIOs, based on the direct measurement of tau and its derivatives will be explored.

Second, the integration of a PT-BAT pilot model into the IAR330 Puma model in a roll step manoeuvre has been carried out by ONERA. The next step will be the prediction of the stability of the closed-loop system by using nonlinear analysis tools.
4. References


5. List of Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>AUG</td>
<td>Augmented Aircraft Model</td>
</tr>
<tr>
<td>BAT</td>
<td>Boundary Avoidance Tracking</td>
</tr>
<tr>
<td>DLR</td>
<td>Deutsches Zentrum für Luft- und Raumfahrt (German Aerospace Center)</td>
</tr>
<tr>
<td>FCS</td>
<td>Flight Control System</td>
</tr>
<tr>
<td>GARTEUR</td>
<td>Group for Aeronautical Research and Technology in Europe</td>
</tr>
<tr>
<td>HUD</td>
<td>Head-Up-Display</td>
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<tr>
<td>NRMSE</td>
<td>Normalised Root-Mean-Square Error</td>
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<td>ONERA</td>
<td>The French Aerospace Lab (Office National d'Etudes et de Recherches Aérospatiales)</td>
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<td>PIO</td>
<td>Pilot Induced Oscillations</td>
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<td>PIO Rating</td>
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<td>Rotorcraft Pilot Couplings</td>
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<td>WP</td>
<td>Work Package</td>
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<td>UoL</td>
<td>University of Liverpool</td>
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6. Appendix A

According to tau-theory, to decelerate successfully to a stop at a goal, the control strategy is to maintain a constant rate of change of tau (\( \dot{\tau} \)) with time as shown in Eq. (A1) [7].

\[
\dot{\tau}_y = 1 - \frac{y \ddot{y}}{\dot{y}^2} = 1 - \frac{\dot{r}_y}{\dot{r}_y} \tag{A1}
\]

where \( \dot{\tau}_y = \frac{y}{\dot{y}} \) and \( \dot{r}_y = \frac{\ddot{y}}{\dot{y}} \)

in which \( y \) is the motion gap (negative), \( \dot{y} \) is the gap closure rate, and \( \ddot{y} \) is the gap closure acceleration. A \( \dot{\tau} \) constant motion can therefore be modelled as either an intrinsic tau-guide (\( \tau_g \)) following strategy,

\[
\tau_y = k \tau_g \tag{A2}
\]

where \( \tau_g = -\frac{1}{2} (1 - \dot{\tau}) \) and \( \dot{\tau} \) provided that the guide takes the form of a constant deceleration, or with an extrinsic guide, the pilot coupling the motion and its velocity. The five situations shown in Table A1 are possible for such motion-gap closures.
<table>
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<th>$\dot{t}$</th>
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<td>$k &gt; 2$</td>
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<tr>
<td>$\dot{t} = 1$</td>
<td>$k = 2$</td>
<td>Constant velocity flight towards the goal/boundary</td>
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<td>$1 &lt; k &lt; 2$</td>
<td>Decelerating flight stopping at the boundary with maximum deceleration at the boundary</td>
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<td>$k = 1$</td>
<td>Constant deceleration flight stopping at the boundary</td>
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<td>$0 &lt; \dot{t} &lt; 0.5$</td>
<td>$0 &lt; k &lt; 1$</td>
<td>Decelerating flight stopping at the boundary with maximum deceleration early in manoeuvre</td>
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